Exercise 30

For the following exercises, solve the equations over the complex numbers.

$$x^2 + 2x + 5 = 0$$

Solution

The two terms with x, x^2 and $2x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 2$, which means $B = 1$ and $B^2 = 1$. Add and subtract 1 from the left side and apply the identity.

$$(x^2 + 2x + 1) + 5 - 1 = 0$$

$$(x + 1)^2 + 4 = 0$$

Now that x appears in only one place, it can be solved for. Subtract 4 from both sides.

$$(x + 1)^2 = -4$$

Take the square root of both sides.

$$\begin{aligned}\sqrt{(x+1)^2} &= \sqrt{-4} \\ &= \sqrt{4(-1)} \\ &= \sqrt{4}\sqrt{-1} \\ &= 2i\end{aligned}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x + 1$.

$$|x + 1| = 2i$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x + 1 = \pm 2i$$

Subtract 1 from both sides.

$$x = -1 \pm 2i$$

Therefore, $x = \{-1 - 2i, -1 + 2i\}$.